



Extension of conventional MHD equilibrium theory to model the fast particle effects

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**5th ITER International Summer School
Aix en Provence, France, June 20 - 24, 2011**

Motion of a single particle

$$m_p \frac{d\mathbf{v}_p}{dt} = \mathbf{F} = q_p (\mathbf{E} + \mathbf{v}_p \times \mathbf{B})$$

depends on the electric and magnetic fields **E** and **B** created by all other particles and external sources

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \mathbf{j}(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} dV'$$

**In theory of tokamaks and stellarators,
the bulk plasma is
most frequently considered
as a continuous medium
described by the
single-fluid MHD equations**

Is it always good? We consider some other options.

Standard MHD equations

Force balance: $\rho d\mathbf{v} / dt = -\nabla p + \mathbf{j} \times \mathbf{B}$ with

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \quad \Rightarrow \quad \text{in equilibrium} \quad \nabla p = \mathbf{j} \times \mathbf{B}$$

Maxwell eqns: $\nabla \cdot \mathbf{B} = 0$, $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$, $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

& sometimes $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \quad \Rightarrow \quad \text{magnetic flux conservation}$

Currents in the equilibrium plasma

With

$$\nabla p = \mathbf{j} \times \mathbf{B}$$

in equilibrium

we have,

$$\mathbf{j}_{\perp} = \frac{\mathbf{B} \times \nabla p}{B^2},$$

$$\mathbf{j} = \mathbf{j}_{\perp} + \mathbf{j}_{\parallel} \text{ and}$$

$$\nabla \cdot \mathbf{j} = 0$$

\Rightarrow

$$\nabla \cdot \mathbf{j}_{\parallel} = -\nabla \cdot \mathbf{j}_{\perp} = \frac{\mathbf{B} \times \nabla p \cdot \nabla B^2}{B^4}$$

Find \mathbf{j}_{\parallel} and solve $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$ (with $\nabla \cdot \mathbf{B} = 0$)

Alternative: kinetic approach

Boltzmann eq:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\mathbf{F}}{m_p} \cdot \nabla_{\mathbf{v}} f = \left(\frac{\delta f}{\delta t} \right)_{coll}$$

when averaged:

$$\frac{\partial}{\partial t} \rho \mathbf{v} + \nabla \cdot \vec{\Gamma} = \mathbf{f}$$

with

$$\vec{\Gamma} = \rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + \frac{\mathbf{B}^2}{2} \vec{\mathbb{1}} + \vec{p}$$

and

$$\vec{p} = \rho_i \langle \mathbf{u}_i \mathbf{u}_i \rangle + \rho_e \langle \mathbf{u}_e \mathbf{u}_e \rangle$$

Distribution of **fast ions** produced by additional heating systems

“is **strongly anisotropic**,

with the NBI produced **fast ions** flowing predominantly **parallel** to the magnetic field, and the ICRH **accelerated ions** characterized by large **perpendicular** energy and mostly trapped orbits”

Fasoli A., et al., Nucl. Fusion **47** S264 (2007).
‘Progress in the ITER Physics Basis’
Chapter 5: Physics of energetic ions

With **such fast ions** $\vec{p} \neq p\vec{\mathbf{I}}$ and $\nabla \cdot \vec{p} \neq \nabla p$

Then we assume

$$\vec{p} = p_{\parallel} \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} + p_{\perp} \left(\vec{\mathbf{I}} - \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} \right),$$

the most simple form of the pressure tensor with anisotropy.

$$(p_{\parallel}, p_{\perp}) = \sum m_p \int (v_{\parallel}^2, \frac{v_{\perp}^2}{2}) f d\mathbf{v}_p,$$

parallel and perpendicular pressures

From isotropic to anisotropic equil.

Instead of $\nabla p = \mathbf{j} \times \mathbf{B}$ in equilibrium

we have $\nabla \cdot \vec{p} = \mathbf{j} \times \mathbf{B}$ with

$$\vec{p} = p_{\parallel} \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} + p_{\perp} \left(\vec{\mathbf{I}} - \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} \right)$$

There is also $\mathbf{j} = \sum q_p \int \mathbf{v}_p f d\mathbf{v}_p$, but ...

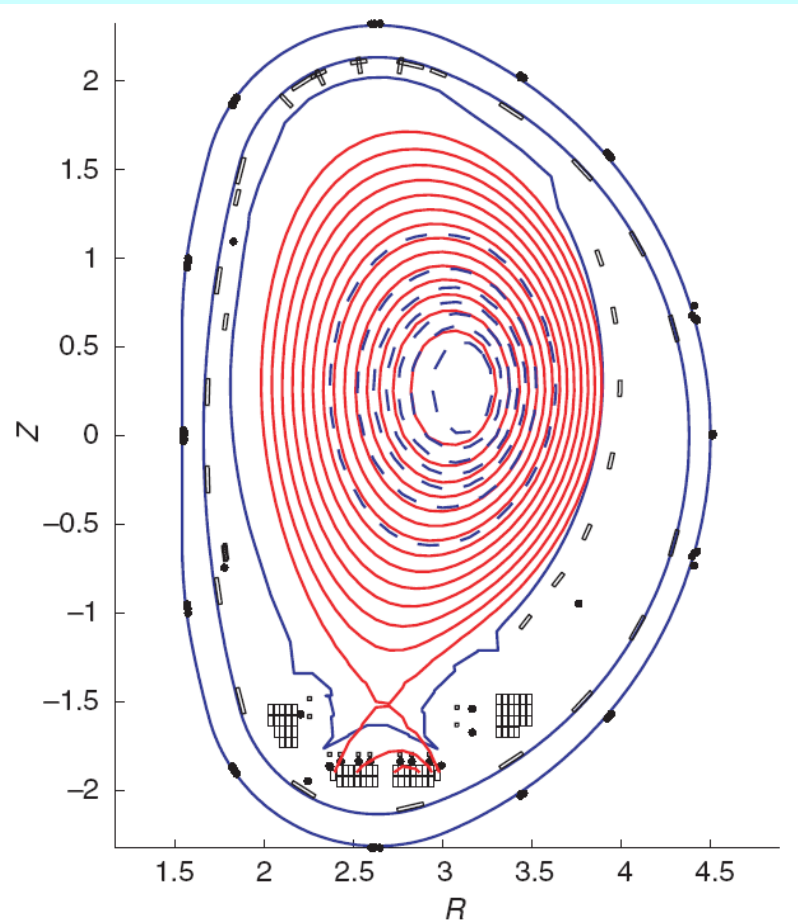
With fast particles, p_{\parallel} and p_{\perp} can be different. What consequences?

To what extent $p_{\parallel} \neq p_{\perp}$?

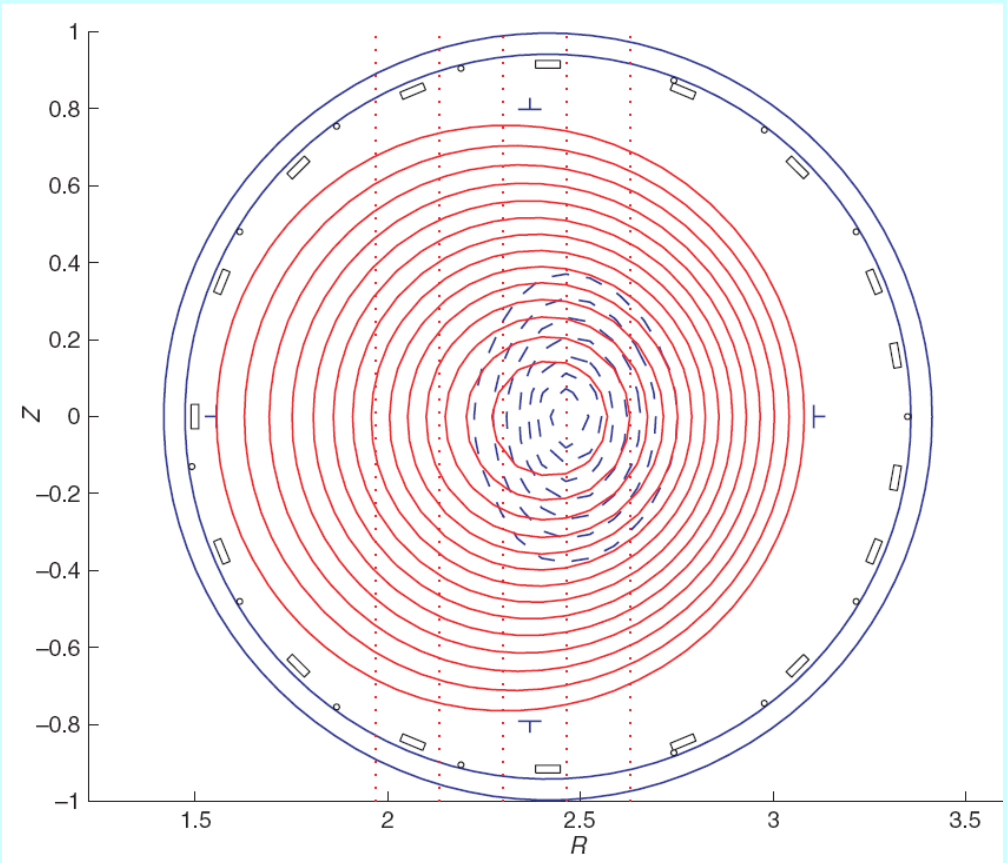
How can we prescribe p_{\parallel} and p_{\perp} ?

Should we develop new theory?

Examples from Zwingmann et al 2001 *PPCF* 43 1441



JET: p_{\parallel} and $a = \text{const}$



Tore Supra: p_{\perp} and $a = \text{const}$

General relations

Start from general equilibrium equations

$$\nabla \cdot \vec{p} = \mathbf{j} \times \mathbf{B}, \quad \mu_0 \mathbf{j} = \nabla \times \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0,$$

$$\vec{p} = p_{\parallel} \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} + p_{\perp} \left(\vec{\mathbf{I}} - \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} \right).$$

As a result we have

$$\nabla p_{\parallel} = \sigma_{\parallel} \nabla (\mathbf{B}^2 / 2) + \mathbf{K} \times \mathbf{B}$$

with $\mu_0 \mathbf{K} = \nabla \times (\sigma \mathbf{B})$, $\sigma = 1 - \sigma_{\parallel}$, and $\sigma_{\parallel} = \frac{p_{\parallel} - p_{\perp}}{\mathbf{B}^2}$

Most popular assumptions

$$p_{\parallel} = p_{\parallel}(a, B), \quad p_{\perp} = p_{\perp}(a, B)$$

with $a = \text{const}$ the flux coordinate: $\mathbf{B} \cdot \nabla a = 0$

1. Good for symmetry (tokamaks),
2. Corresponds to the leading order solution of the Fokker–Planck equation for the distribution function f (which is $\mathbf{B} \cdot \nabla f = 0$ in this case)

Other models? Better choice of p_{\parallel} and p_{\perp} ?

Examples of p_{\parallel} and p_{\perp} prescription

Zwingmann W, Eriksson L G and Stubberfield P, **Equilibrium analysis of tokamak discharges with anisotropic pressure**, 2001 *Plasma Phys. Control. Fusion* **43** 1441

$$P'_{\parallel} = p'_i(\bar{\Psi}) + p'_a(\bar{\Psi}, R) = \sum_{k=1}^{\text{NP}} c_k g_k(\bar{\Psi}; 1) + \sum_{k=1}^{\text{NP}} \sum_{n=1}^{\text{NA}} c_{k+n * \text{NP}} f_n(\bar{r}) g_k(\bar{\Psi}; \delta).$$

$$P_{\perp}(\Psi, R) = P_{\parallel}(\Psi, R) + R \frac{\partial P_{\parallel}(\Psi, R)}{\partial R}.$$

“The present analysis was carried out with one anisotropy term”

“contributions from neutral beams and/or RF heating are obtained from suitable power deposition codes”

Examples of p_{\parallel} and p_{\perp} prescription

Cooper W A *et al* 2005, Three-dimensional **anisotropic** pressure equilibria that model balanced tangential neutral beam injection effects, *Plasma Phys. Control. Fusion* **47** 561

$$F(s, E, \mu) = \frac{h(s)}{E^{3/2} + E_c^{3/2}} \left[1 - \frac{\mu B_m(s)}{E} \right]^L.$$

$$p_{b\perp}(s, B) = p_{b\parallel}(s, B) - B \left. \frac{\partial p_{b\parallel}}{\partial B} \right|_s$$

“modified slowing down distribution”

“model the effects of balanced tangential neutral beam injection”

Examples of p_{\parallel} and p_{\perp} prescription

Cooper W A *et al* 2006 **Anisotropic pressure bi-Maxwellian distribution function model for three-dimensional equilibria** *Nucl. Fusion* **46** 683

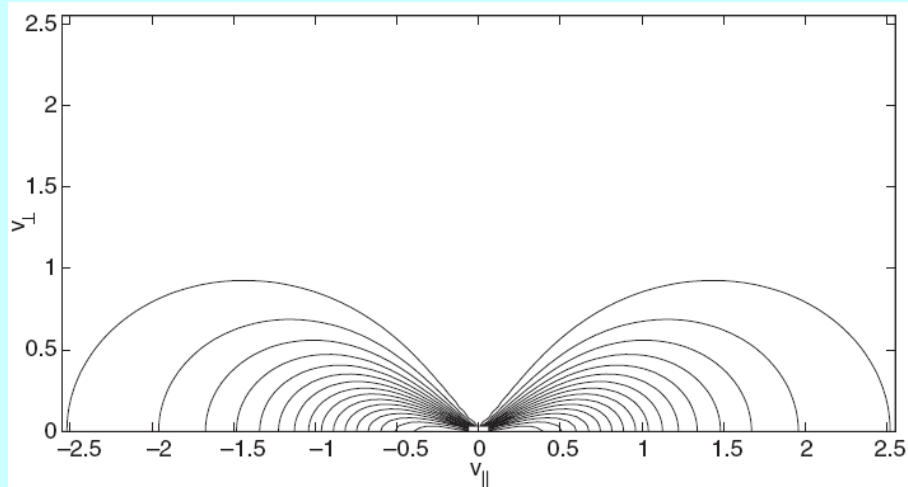
$$\mathcal{F}_h(s, \mathcal{E}, \mu) = \mathcal{N}(s) \left(\frac{m_h}{2\pi T_{\perp}(s)} \right)^{3/2} \times \exp \left[-m_h \left(\frac{\mu B_C}{T_{\perp}(s)} + \frac{|\mathcal{E} - \mu B_C|}{T_{\parallel}(s)} \right) \right].$$

$$p_{\perp}(s, B) = p_{\parallel}(s, B) - B \left. \frac{\partial p_{\parallel}}{\partial B} \right|_s$$

“Large parallel and perpendicular anisotropy factors can be explored through the choice of the temperature ratio T_{\parallel}/T_{\perp} ”

Examples: Contours of constant f

model for balanced tangential NBI

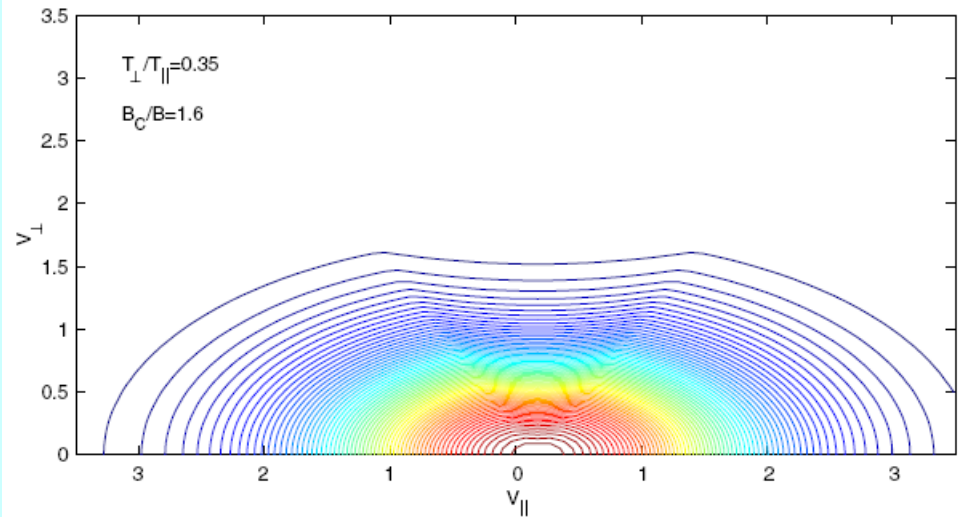
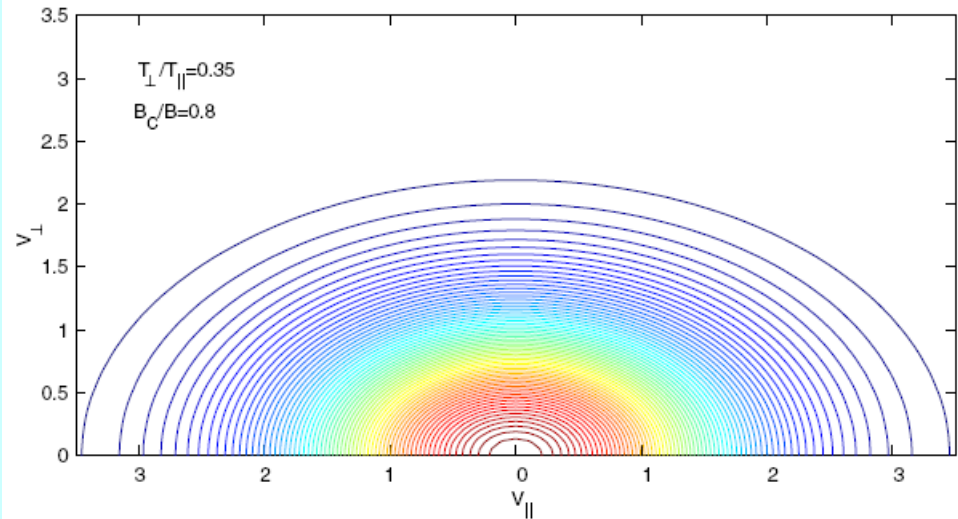


Cooper W A *et al* 2005 *Plasma Phys. Control. Fusion* 47 561

Cooper W A *et al* 2006 *Nucl. Fusion* 46 683

(Bi-Maxwellian) \Rightarrow

function with large parallel anisotropy



Parallel force balance

$$\nabla p_{\parallel} = \sigma_{\parallel} \nabla (\mathbf{B}^2 / 2) + \mathbf{K} \times \mathbf{B} \Rightarrow \mathbf{B} \cdot \nabla p_{\parallel} = \sigma_{\parallel} \mathbf{B} \cdot \nabla (\mathbf{B}^2 / 2)$$

which is equivalent to $\mathbf{B} \cdot \nabla (p_{\parallel} + p_{\perp}) = -\mathbf{B}^2 \mathbf{B} \cdot \nabla \sigma_{\parallel}$

We have $\mathbf{B}^2 = \mathbf{B}_0^2 + (\mathbf{B}^2 - \mathbf{B}_0^2)$ with $|\mathbf{B}^2 / \mathbf{B}_0^2 - 1| \ll 1$ in tokamaks and stellarators. Then $p_{\parallel} + p_{\perp} + \mathbf{B}_0^2 \sigma_{\parallel} = C(a)$.

$$p_{\parallel} \left(1 + \frac{\mathbf{B}_0^2}{\mathbf{B}^2} \right) + p_{\perp} \left(1 - \frac{\mathbf{B}_0^2}{\mathbf{B}^2} \right) = 2p_{\parallel 0} + \delta$$

Parallel force balance: consequences

$$p_{\parallel} \approx p_{\parallel 0} + \frac{p_{\parallel 0} - p_{\perp}}{2} \left(1 - \frac{\mathbf{B}_0^2}{\mathbf{B}^2} \right) \quad \text{with } p_{\parallel 0} = p_{\parallel 0}(a)$$

\Rightarrow **in tokamaks and stellarators,** $p_{\parallel} - p_{\parallel 0}(a)$
must be small even at large variations of p_{\perp} .

$$p_{\parallel} = p_{\parallel 0} + \tilde{p}_{\parallel}$$

Large \tilde{p}_{\parallel} **can be produced by very large** \tilde{p}_{\perp} **only.**

Some numerical results

Cooper W A *et al* 2005 *Plasma Phys. Control. Fusion* **47** 561

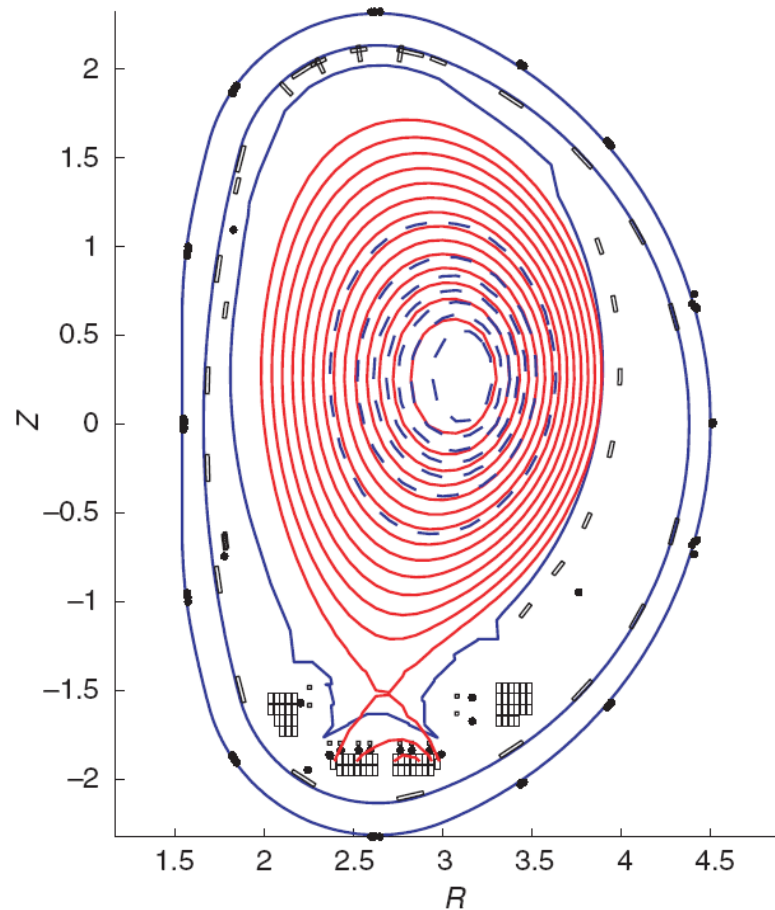
“the total pressure surfaces with $p_{\parallel} \gg p_{\perp}$ do *not appear to significantly deviate* from the flux surfaces which is **in stark contrast** to earlier results with $p_{\perp} \gg p_{\parallel}$ where the pressure surfaces can become *completely decoupled* from the flux surfaces”

Jucker M *et al* 2008 *Plasma Phys. Control. Fusion* **50** 065009

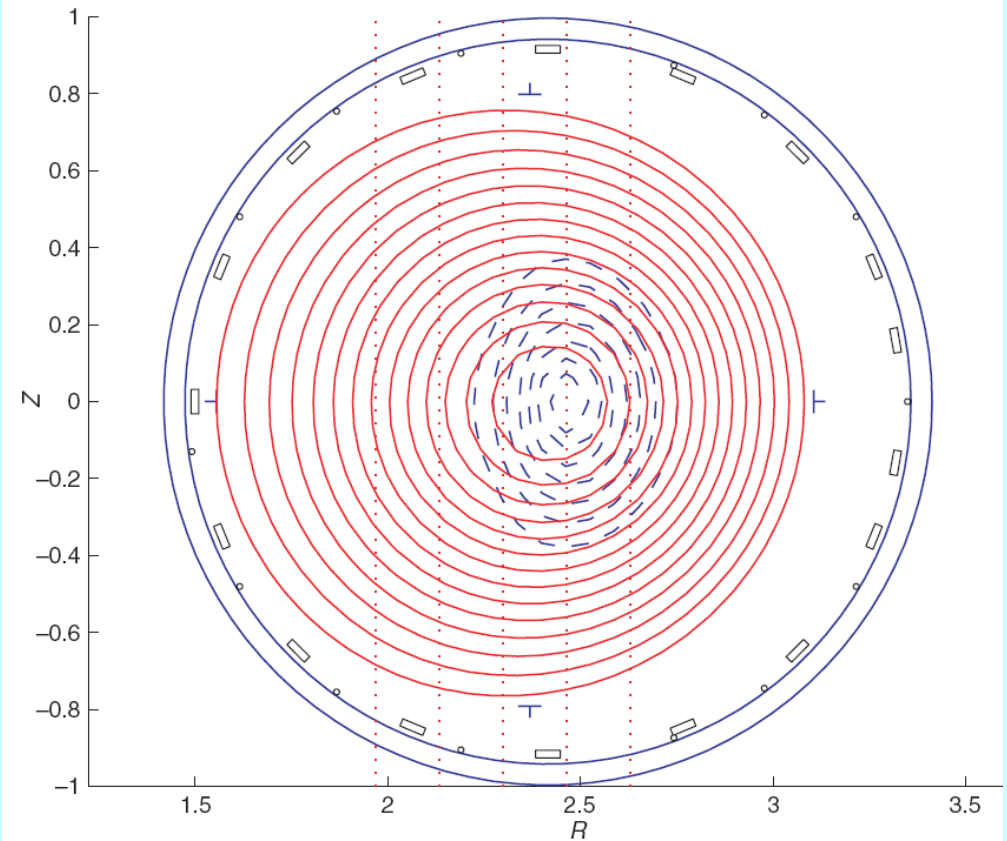
“*Significant differences* between parallel and perpendicular pressure anisotropy are observed.”

“poloidal variation in p_{\parallel} is only *non-negligible* when $p_{\perp} \gg p_{\parallel}$ ”

Examples from Zwingmann et al 2001 *PPCF* 43 1441



JET: p_{\parallel} and $a = \text{const}$



Tore Supra: p_{\perp} and $a = \text{const}$

Perpendicular force balance

$$\nabla p_{\parallel} = \sigma_{\parallel} \nabla (\mathbf{B}^2 / 2) + \mathbf{K} \times \mathbf{B}$$

with $\mu_0 \mathbf{K} = \nabla \times (\sigma \mathbf{B}) \Rightarrow$

$$\mathbf{j}_{\perp} = \frac{\mathbf{B}}{\sigma \mathbf{B}^2} \times \left(\nabla p_{\perp} + \frac{p_{\parallel} - p_{\perp}}{\mathbf{B}^2} \nabla \frac{\mathbf{B}^2}{2} \right), \text{ mainly determined by } p_{\perp}.$$

After some algebra (cylinder):

$$2 \frac{\Delta \Phi}{\Phi_0} = \frac{B_J^2}{B_0^2} - \bar{\beta}_{\perp} + 2 \frac{\Delta \Phi_{st}}{\Phi_0},$$

where $\Delta \Phi = \int_{S_{\perp}} (\mathbf{B} - \mathbf{B}_v) d\mathbf{S}_{\perp}$ is the diamagnetic signal.

Equilibrium current, *general*

$$\nabla \cdot \mathbf{K}_{\parallel} = -\nabla \cdot \mathbf{K}_{\perp} = \frac{\mathbf{B} \times \nabla(p_{\parallel} + p_{\perp})}{2\sigma \mathbf{B}^4} \cdot \nabla(\mathbf{B}^2 + 2p_{\perp})$$

with $\sigma \mathbf{j} = \mathbf{K} + \nabla \sigma_{\parallel} \times \mathbf{B} / \mu_0$ and $\sigma = 1 - (p_{\parallel} - p_{\perp}) / \mathbf{B}^2 \approx 1$

If $\nabla(\mathbf{B}^2 + 2p_{\perp})$ could be replaced by $\nabla \mathbf{B}^2$, we would obtain \mathbf{K} (and \mathbf{j}) depending on $p_{\parallel} + p_{\perp}$.

Therefore, p_{\perp} is a key function

Equilibrium current, *simplified*, \mathbf{j}_{\parallel}

$$\nabla \cdot \mathbf{K}_{\parallel} = -\nabla \cdot \mathbf{K}_{\perp} = \frac{\mathbf{B} \times \nabla(p_{\parallel} + p_{\perp})}{2\sigma\mathbf{B}^4} \cdot \nabla(\mathbf{B}^2 + 2p_{\perp})$$

With $p_{\parallel} \approx p_{\parallel 0}$ and $|\tilde{p}_{\perp}| \ll \epsilon\mathbf{B}^2$ we have

$$\nabla \cdot \mathbf{j}_{\parallel} \approx \frac{\mathbf{B} \times \nabla(p_{\parallel} + p_{\perp})}{2\sigma\mathbf{B}^4} \cdot \nabla\mathbf{B}^2.$$

Are these conditions satisfied in experiments?

Equil. currents, *simplified*, summary

$$\nabla \cdot \vec{p} = \mathbf{j} \times \mathbf{B} \quad \text{with} \quad \vec{p} = p_{\parallel} \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} + p_{\perp} \left(\vec{\mathbf{I}} - \frac{\mathbf{B}\mathbf{B}}{\mathbf{B}^2} \right).$$

Perpendicular: $\mathbf{j}_{\perp} \approx \frac{\mathbf{B} \times \nabla p_{\perp}}{\mathbf{B}^2}$, determined by p_{\perp}

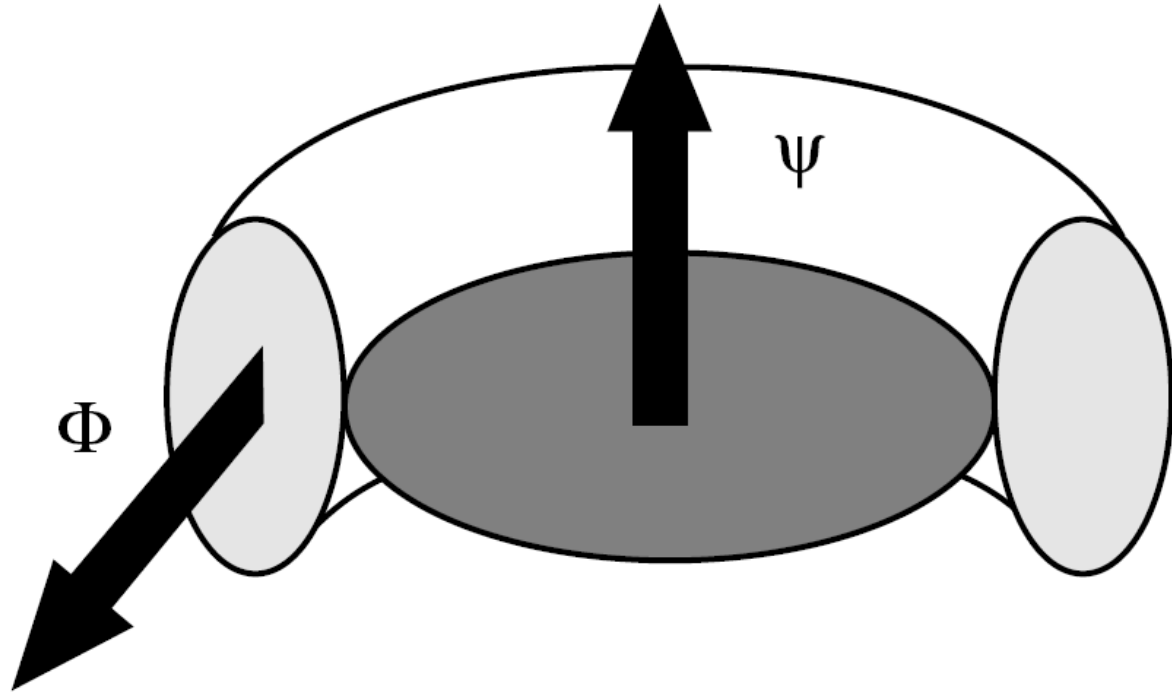
Parallel: $\nabla \cdot \mathbf{j}_{\parallel} \approx \frac{\mathbf{B} \times \nabla (p_{\parallel} + p_{\perp}) \cdot \nabla \mathbf{B}^2}{2\mathbf{B}^4}$, determined by $p_{\parallel} + p_{\perp}$.

Poloidal ψ and toroidal Φ magnetic fluxes associated with a toroidal magnetic surface

$$\psi \equiv \int \mathbf{B} \cdot d\mathbf{S}_{pol}$$

$$F \equiv \int \mathbf{j} \cdot d\mathbf{S}_{pol}$$

$$\Phi \equiv \int \mathbf{B} \cdot d\mathbf{S}_{tor}$$



$$2\pi\mathbf{B} = \nabla\psi \times \nabla\zeta + F\nabla\zeta$$

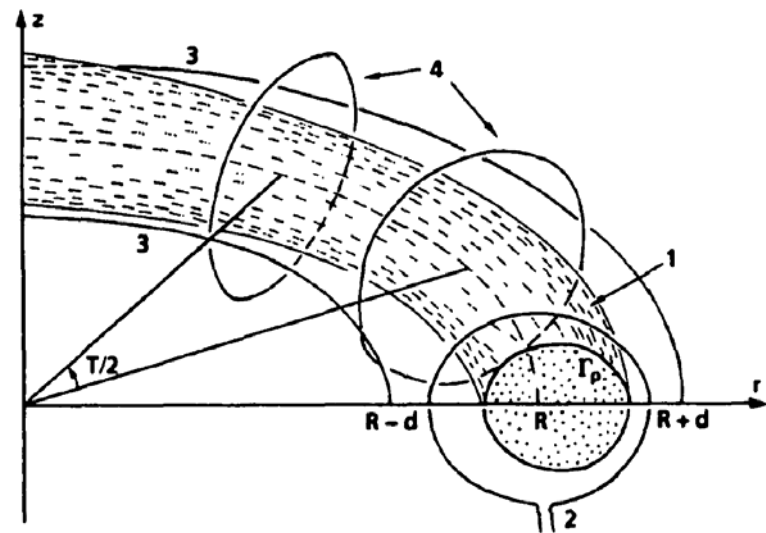
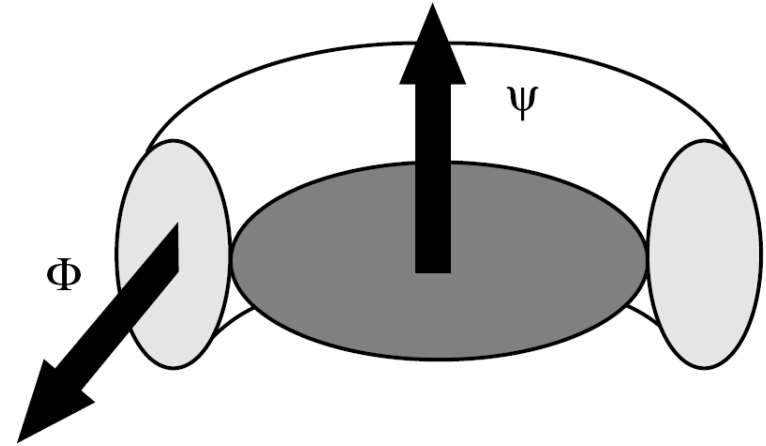
Magnetic diagnostics

ψ is determined by \mathbf{j}_{\parallel} ,

\Rightarrow by $p_{\parallel} + p_{\perp}$

Φ is determined by \mathbf{j}_{\perp} ,

\Rightarrow by p_{\perp}



Experimental Results

Nucl. Fusion 45 (2005) L33–L36

Measurement of anisotropic pressure using magnetic measurements in LHD

T. Yamaguchi¹, K.Y. Watanabe^{1,2}, S. Sakakibara²,
Y. Narushima^{1,2}, K. Narihara², T. Tokuzawa^{1,2}, K. Tanaka²,
I. Yamada², M. Osakabe², H. Yamada^{1,2}, K. Kawahata^{1,2},
K. Yamazaki³ and LHD Experimental Group²

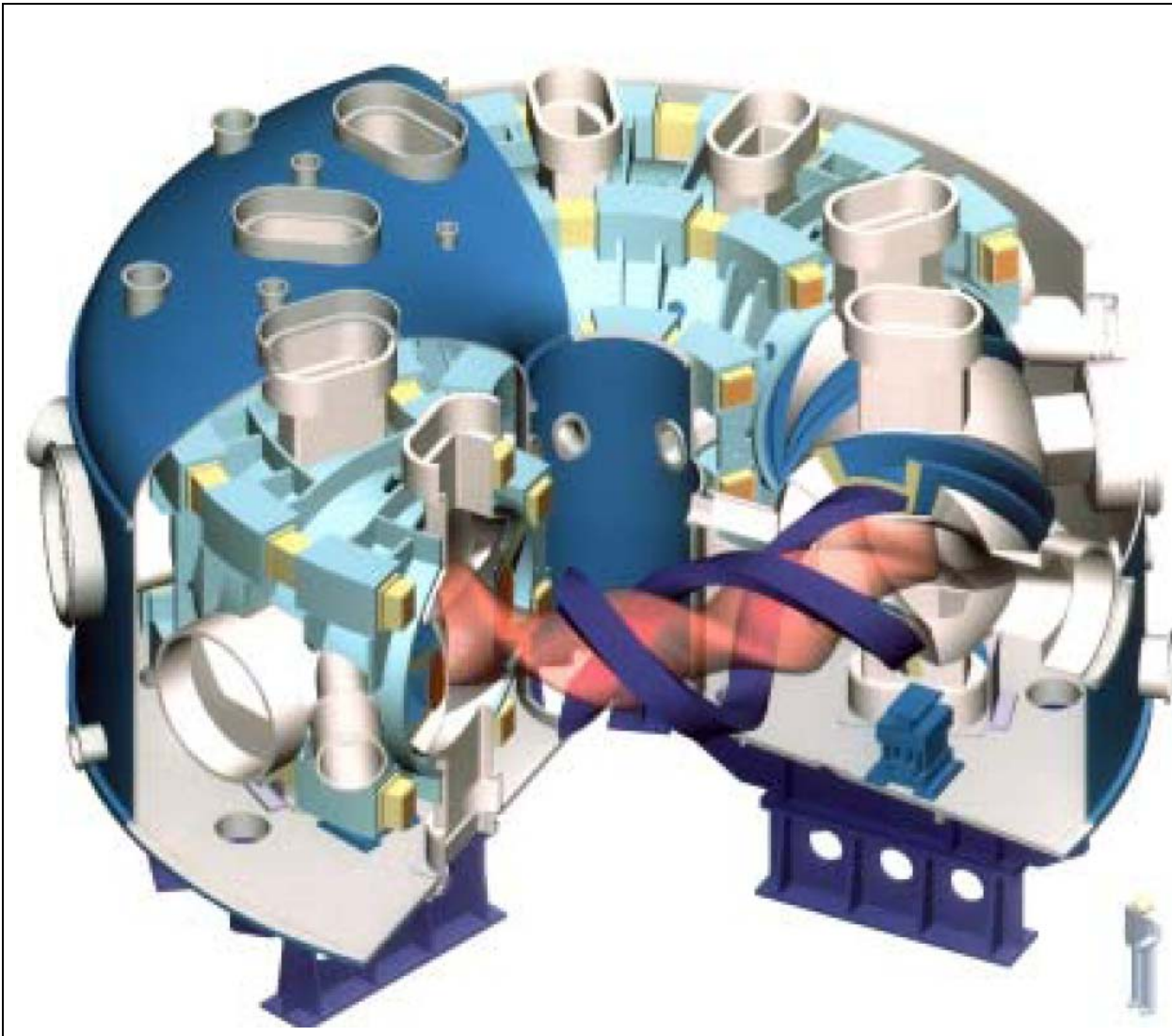


核融合科学
研究会

“In low density discharges of a Large Helical Device (LHD), **anisotropic pressure** is expected because the LHD has powerful tangential neutral beam injection systems.

We show the strong correlation between the **pressure anisotropy** due to the beam pressure based on Monte Carlo calculations and the ratio of the diamagnetic loop signal and the saddle loop signal.”

Large Helical Device (LHD)



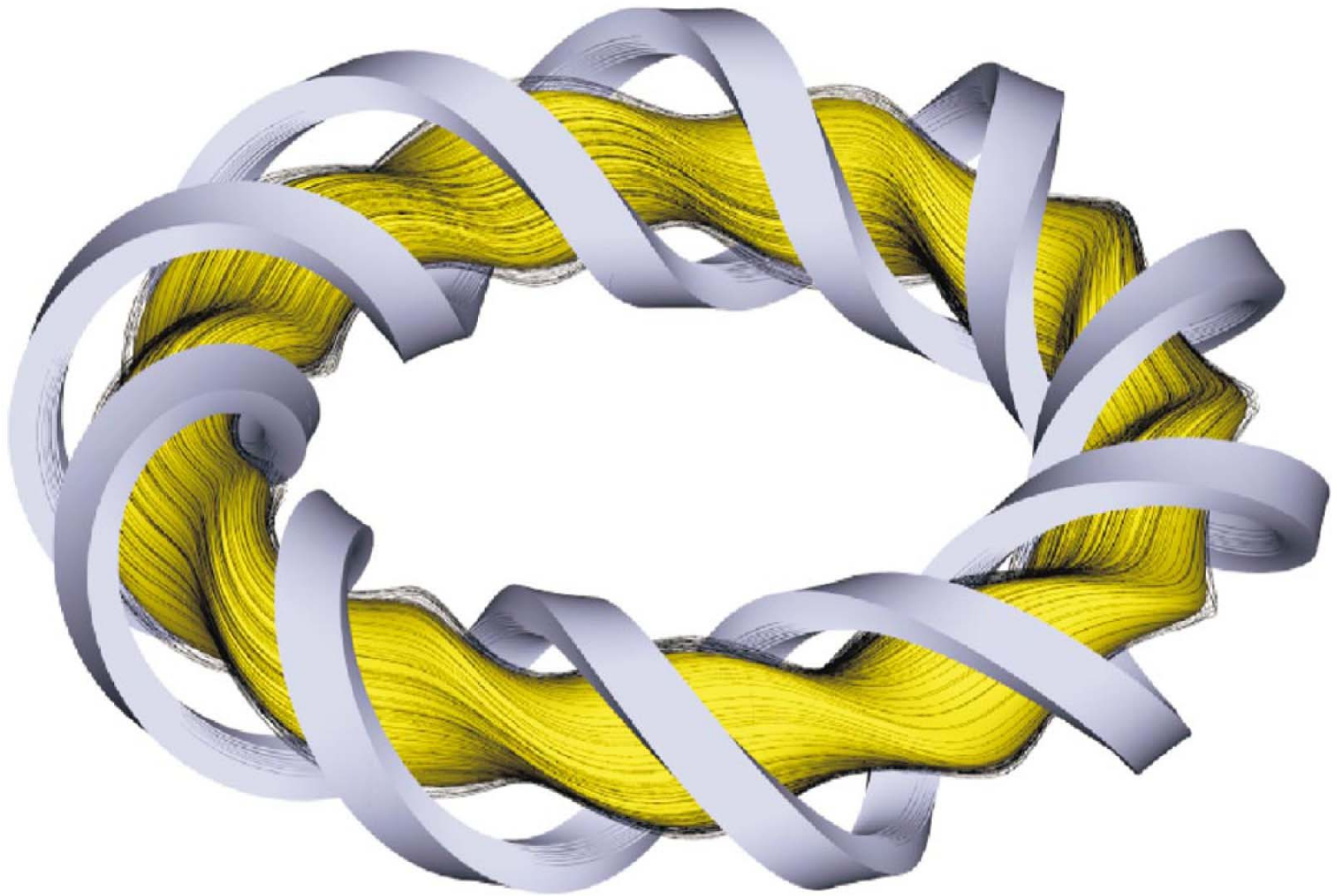
All superconducting coil system

Major radius = 3.42 –4.1 m

Plasma radius = 0.6 m

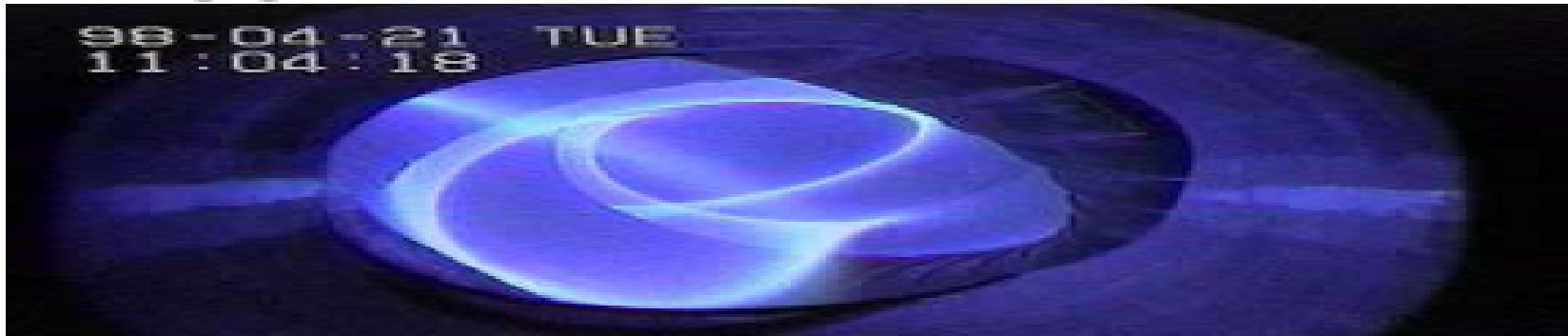
Plasma volume = 30 m³
Toroidal field 2.9 T

(S. Sudo, 2003)



P2-35

19th International Toki Conference (ITC19)
December 8 - 11, 2009 Ceratopia Toki



Anisotropic Pressure Effect on the MHD Equilibrium in LHD

**K.Y. Watanabe, T. Yamaguchi, S. Sakakibara,
Y. Suzuki, Y. Narushima and LHD experiment group**

**National Institute for Fusion Science, 322-6 Oroshi-cho, Toki 509-5292,
Japan**

K.Y. Watanabe, et al., P2-35, ITC-2009

Numerical calculated prediction of anisotropic pressure from the beam

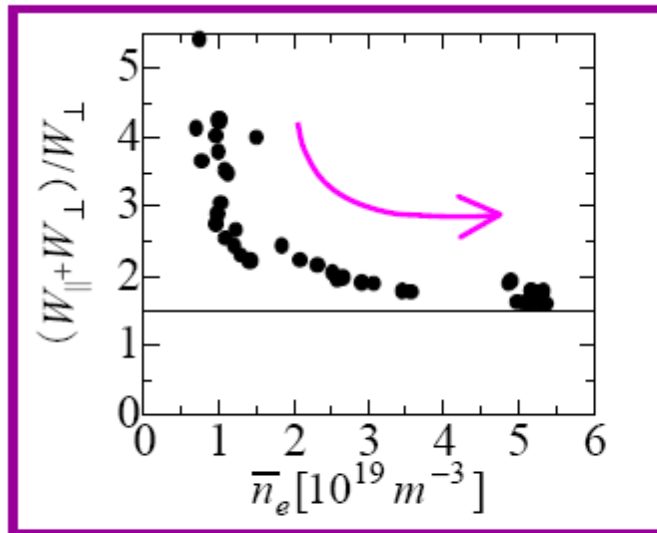
Prediction by FIT code : ※ S. Murakami, N. Nakajima, M. Okamoto, Trans. Fusion Technol., 27, (1995) 256.

The birth profile of fast ion from NBI is estimated by Monte-Carlo simulation

Beam pressure is estimated by the steady state solution of the Fokker-Planck eq.

Direct loss effect is taken into account.

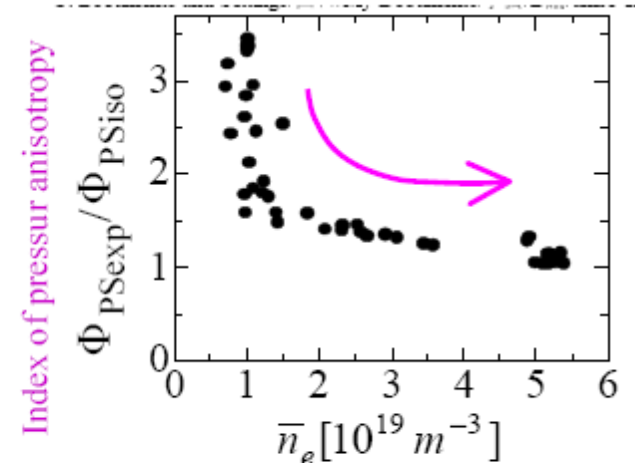
$$\begin{aligned}
 &\bullet \text{FIT code} \rightarrow W_{beam\parallel}, W_{beam\perp} \\
 &\bullet W_{dia} \text{ and FIT code} \rightarrow W_{thermal} \\
 &\quad (W_{dia} = W_{thermal} + (3/2)W_{beam\perp})
 \end{aligned}
 \longrightarrow
 \begin{cases}
 W_{\parallel} = (1/3)W_{thermal} + W_{beam\parallel} \\
 W_{\perp} = (2/3)W_{thermal} + W_{beam\perp}
 \end{cases}$$



Numerical calculated prediction

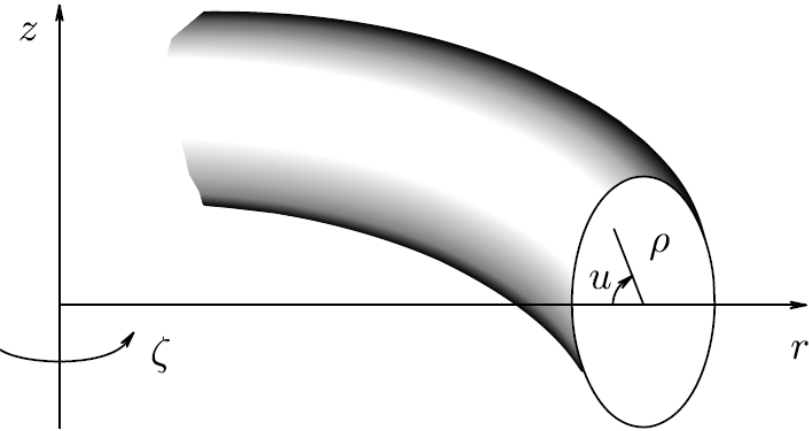


Consistent



Estimation by Magnetics 9

Plasma equilibrium with toroidal rotation



$$\rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla\vec{p} + \mathbf{j}\times\mathbf{B}$$

$$\mathbf{v} = v_t(r,z)\mathbf{e}_\zeta \Rightarrow \rho(\mathbf{v}\nabla)\mathbf{v} = -\frac{\rho v_t^2}{r}\mathbf{e}_r$$

Scalar pressure:

$$\frac{\rho v_t^2}{r}\mathbf{e}_r - \nabla p + \mathbf{j}\times\mathbf{B} = 0$$

Additional force along the major radius

$$\int \frac{p + \rho v_t^2}{r} dV + \int \mathbf{e}_r (\mathbf{j}\times\mathbf{B}) dV = 0$$

Equilibrium with toroidal rotation, estimates

$$\frac{p}{\rho} = v_{T_i}^2 \left(1 + \frac{n_e T_e}{n_i T_i} \right)$$

with

$$v_{T_i} \equiv \sqrt{\frac{T_i}{m_i}}$$

$$\frac{\rho v_t^2}{p} \approx \frac{v_t^2}{2v_{T_i}^2}$$

For hydrogen

$$v_{T_i} = 979 \sqrt{\frac{T_i}{T_0}} \text{ km/s}$$

with

$$T_0 = 10 \text{ keV}$$

Large $-\rho(\mathbf{v}\nabla)\mathbf{v} = \mathbf{e}_r \rho v_t^2 / r$ **at very large velocity only**

Plasma with toroidal rotation, estimates

Proton mass

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

Plasma density

$$n = 10^{20} \text{ m}^{-3}$$

\Rightarrow **mass density**

$$\rho = m_p n = 1.67 \times 10^{-7} \text{ kg/m}^3$$

Compare to

water $\rho = 10^3 \text{ kg/m}^3$

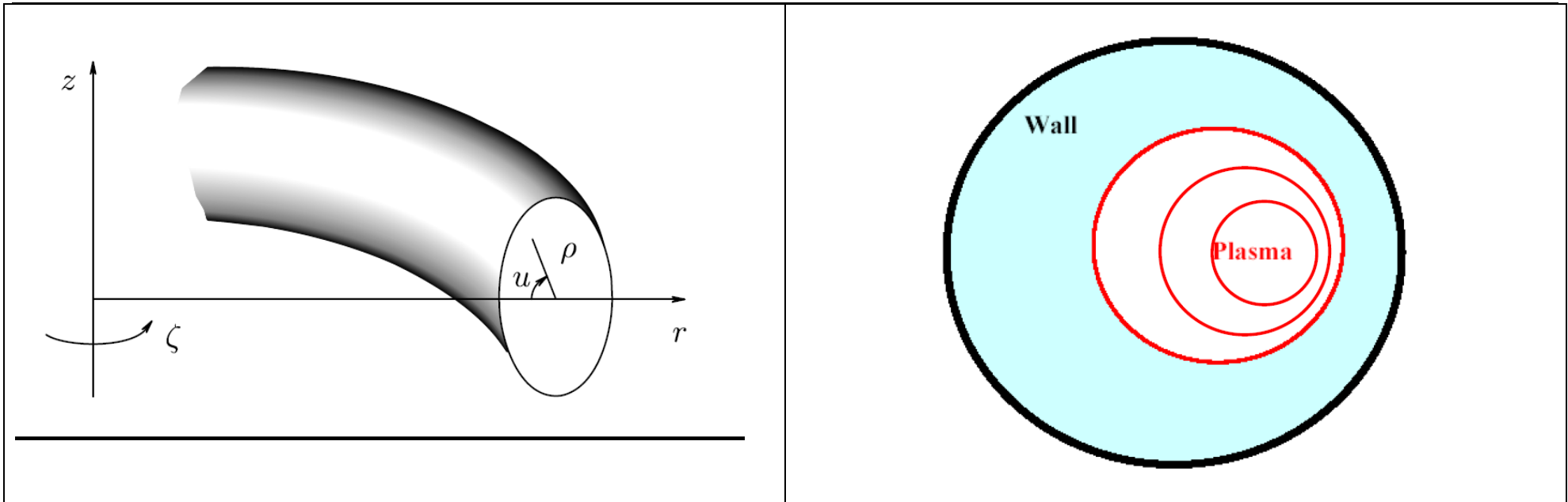
air $\rho = 1.29 \text{ kg/m}^3$

ITER: plasma volume

$$V_{\text{plasma}} = 870 \text{ m}^3$$

Mass of H plasma $M = \rho V_{\text{plasma}} = 1.45 \times 10^{-4} \text{ kg}$

Rotation and Shafranov shift



$$\Delta' = \Delta'_S - \frac{a}{R} \frac{\overline{\rho v_t^2} - \rho v_t^2}{B_\theta^2}$$

with

$$\Delta'_S = -\frac{a}{R} \left[\frac{l_i}{2} + 2 \frac{\overline{p} - p}{B_\theta^2} \right]$$

with

$$l_i \equiv \overline{B_\theta^2} / B_\theta^2$$

and

$$\overline{X} \equiv \frac{2}{a^2} \int_0^a X \rho d\rho$$

Rotation and Shafranov shift - 2

$$\Delta'(b) = -\frac{b}{R} \left[\frac{l_i}{2} + \frac{2\bar{p} + \overline{\rho v_t^2}}{B_J^2} \right]$$

The global effect of toroidal rotation is larger outward shift, but only weak increase

Effect comparable to pressure at $v_t \sim v_{T_i}$,

or $v_{beam} \sim v_{T_i} \frac{S_{plasma}}{S_{beam}}$ for a beam

Summary

➤ **Fast particles create the pressure anisotropy and rotation**

➤ **In equilibrium, the deviations from conventional MHD must be mainly related to p_{\perp}**

$$p_{\parallel} \left(1 + \frac{\mathbf{B}_0^2}{\mathbf{B}^2} \right) + p_{\perp} \left(1 - \frac{\mathbf{B}_0^2}{\mathbf{B}^2} \right) \approx 2p_{\parallel 0}$$

➤ **In some cases it must be possible to estimate the degree of **pressure anisotropy** by magnetic measurements**

\mathbf{j}_{\perp} is determined by p_{\perp} , while \mathbf{j}_{\parallel} is \sim determined by $p_{\parallel} + p_{\perp}$

➤ **Reliable when $p_{\perp} \ll p_{\parallel}$ or $p_{\perp} \approx p_{\perp 0}(a)$**

-
- **Toroidal rotation gives slightly larger Shafranov shift, but a strong effect is only at very large speed**
-

For more details see

1. Pustovitev V.D., **Equilibrium of Rotating and Nonrotating Plasmas in Tokamaks**, *Plasma Physics Reports* **29** (2003) p. 105.

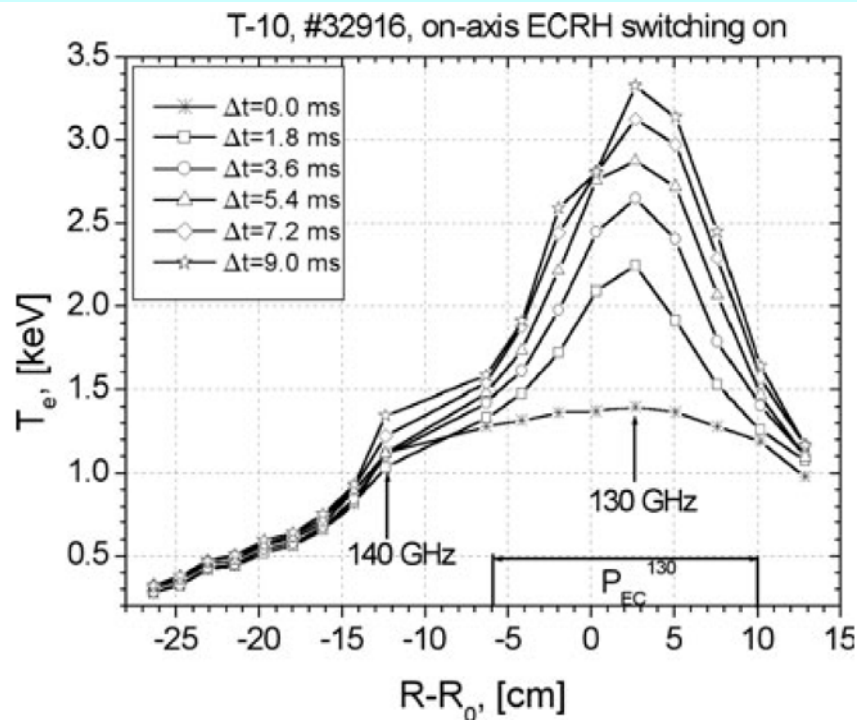
2. Pustovitev V.D., **Anisotropic pressure effects on plasma equilibrium in toroidal systems**, *Plasma Phys. Control. Fusion* **52**, 065001 (2010).

and references therein

Backup slides

Experiments on T-10

V.F. Andreev, et al., “*The ballistic jump of the total heat flux after ECRH switching on in the T-10 tokamak*” Plasma Phys. Control. Fusion **46**, 319 (2004).



Electron temperature profiles for several time slices after on-axis ECRH switching on (shot #32916, input on-axis ECRH power ~ 600 kW)

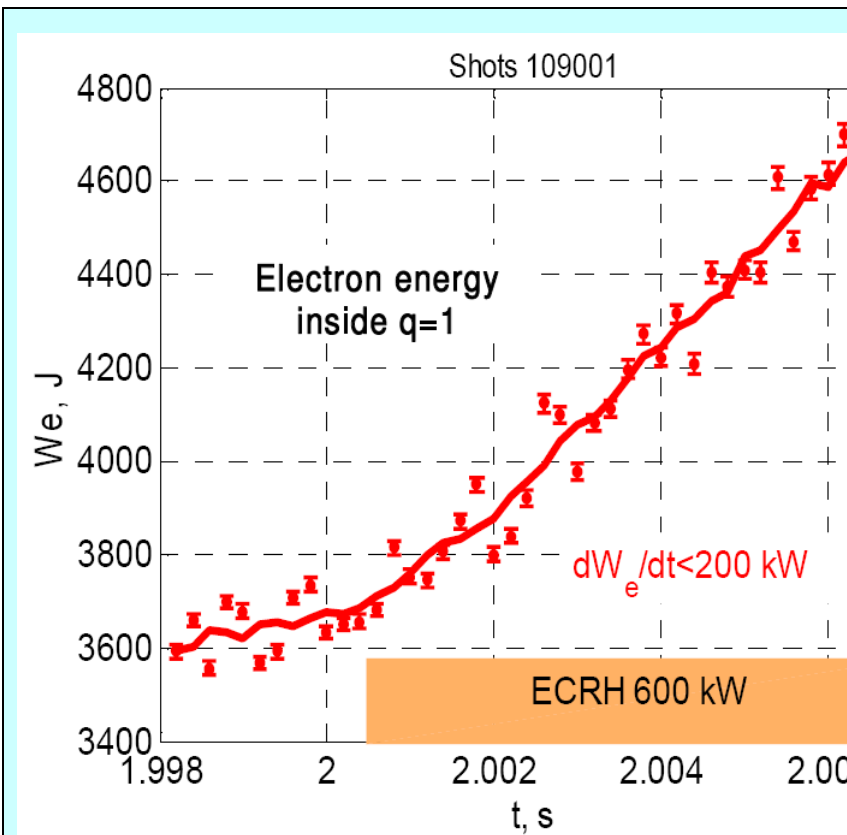
“in the heating region plasma can ‘assimilate’ **only part** of the input power”

“**up to 60%** of ECRH power is rapidly thrown out of the plasma core to the peripheral plasma (‘ballistic effect’)”

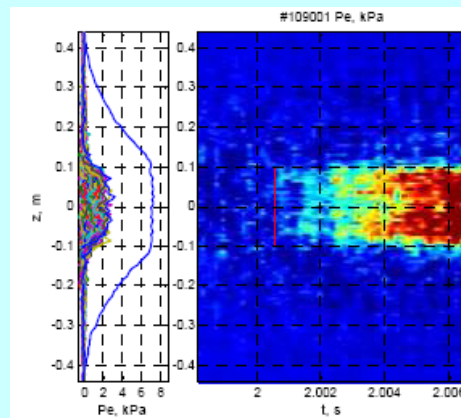
“effective heat diffusivity **increases** up to values of $10\text{--}15\text{ m}^2\text{ s}^{-1}$ in the first $100\text{--}200\text{ }\mu\text{s}$ and **decreases** down to values of $1.5\text{--}2.0\text{ m}^2\text{ s}^{-1}$ during the following $1\text{--}2\text{ ms}$.”

Experiments on TEXTOR

M.Yu. Kantor, et al., “Thomson scattering diagnostic for study fast events in the TEXTOR plasma” 36th EPS Conf. Plasma Phys. (Sofia, June 29 – July 3 2009) P-1.184



Electron heating inside $q=1$ surface during ECRH



“the absorbed energy is **perfectly confined** inside the $q=1$ surface during the first 5 ms”

“the electron heating rate inside the $q=1$ surface calculated from the local TS data **shows ~200 kW** which is

only one third of the launched EC power”

Equations for the heat transport in

V.F. Andreev, et al., Plasma Phys. Control. Fusion **46**, 319 (2004).

$$\frac{3}{2} \frac{\partial}{\partial t} (nT) = -\frac{1}{r} \frac{\partial}{\partial r} rW + P_{OH} + Q + P_{EC}$$

$$W = -\chi_e \nabla T + nT \vec{u}$$

P_{OH} the Ohmic power, Q other heat sources,

P_{EC} the ECRH power. Later \vec{u} is disregarded

“the relative density variation is much less than the relative temperature variation”

No electromagnetic interaction here

Our main equations **in**

[3] V.D. Pustovitov and S.A. Stepanyan, Plasma Phys. Control. Fusion **53**, 035004 (2011). [4] V.D. Pustovitov, Plasma Phys. Rep. **37**, 109 (2011).

Force balance: $\rho d\mathbf{v} / dt = -\nabla p + \mathbf{j} \times \mathbf{B} \Rightarrow \nabla p = \mathbf{j} \times \mathbf{B}$

Energy balance: $\frac{\partial}{\partial t} \left(\frac{3}{2} p + \frac{\mathbf{B}^2}{2} \right) + \nabla \cdot \left(\frac{5}{2} p \mathbf{v} + \mathbf{E} \times \mathbf{B} + \mathbf{q}_1 \right) = s,$

Maxwell eqns: $\nabla \cdot \mathbf{B} = 0,$ $\nabla \times \mathbf{B} = \mathbf{j},$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ &

$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \Rightarrow$ **magnetic flux conservation:**

$\Phi_{pl} \equiv \int_{plasma} \mathbf{B} \cdot d\mathbf{S}_{\perp} = inv$ **plasma,** $\Phi_e \equiv \int_{gap} \mathbf{B} \cdot d\mathbf{S}_g = inv$ **plasma-wall gap**

Integral energy balance

Integrate $\frac{\partial}{\partial t} \left(\frac{3}{2} p + \frac{\mathbf{B}^2}{2} \right) + \nabla \cdot \left(\frac{5}{2} p \mathbf{v} + \mathbf{E} \times \mathbf{B} + \mathbf{q}_1 \right) = s$ up to the wall ($\mathbf{E} \times \mathbf{n} = 0$):

$$\frac{d}{dt} \int_{\text{plasma}} \frac{3}{2} p dV = \int_{\text{plasma}} p_{in} dV - \frac{d}{dt} \int_{\text{total}} \frac{\mathbf{B}^2}{2} dV$$

last term - “missing power”

The magnetic energy change is small, $\delta W_m^{pl} + \delta W_m^{gap} \approx 0$.

The heating power goes to the plasma,
no “missing power” at fast processes.